8 - Probability 2

Examples. What's wrong with each situation?

(a) A printer makes 0, 1, or 2 mistakes printing a document are 0.20, 0.35, -0.25, 0.70.

(b) In (a): probabilities are 0.10, 0.10, 0.30, 0.70.

(c) The probability it rains today is 0.40; The probability it does not rain today is 0.52.

(d) Draw a card: the probability the card is both heart and black is 1/8.

Example 2. Two dice are rolled. Find the probability at least one die roll is ≥ 3.

Answer 1: Let A = {1st die roll is ≥ 3} and B = {2nd die roll is ≥ 3}:

P(A) = 4/6 (since 2nd roll does not affect the 1st roll)

P(B) = 4/6 (since 1st roll does not affect the 2nd roll)

P(A∩B) = P(both rolls are ≥ 3) = (4x4)/(6x6)

So P(A or B) = P(A∪B) = 2/3 + 2/3 - 4/9 = 8/9.

Answer 2 (Complement):

1 - P(both rolls are 2 or less) = 1 - (2x2)/(6x6) = 1-1/9 = 8/9.

Example 3. The probability a poker hand of 5 cards has at least one red card:

Answer: = 1 - P(5-card hand has no red card) = 1 - (26 choose 5) / (52 choose 5)

Exercise.

1. Find ?:

Language spoken by Canadians:	English	French	Asian/Pacific	Other
Probability:	0.63	0.22	0.06	?

2. The probability that an American industry will be located in Shanghai, China, is 0.6, the probability that it will be located in Beijing, China, is 0.4, and the probability that it will be located in either Shanghai or Beijing or both is 0.9. What is the probability that the industry will be located: (a) in both cities? (b) in neither city?

3. If each coded item in a catalog begins with 2 distinct letters followed by 3 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit even.

4. In a poker hand consisting of 6 cards, find the probability of holding
(a) 3 aces. (b) 4 hearts and 2 clubs. (c) 4 hearts or 2 clubs.

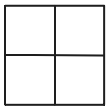
5. Of 100 students, 54 studied math, 69 studied history, and 35 studied both math and history. If one of these students is selected at random, find the probability that
(a) the student took math or history;
(b) the student did not take either of these subjects;
(c) the student took history but not math.

6. A raffle randomly selects two separate winners from Ann, Bob, Cass, Dan, and Em. Each has the same chance of winning. Find the probability Dan does not win.

Answers:
1. 0.09
2. (a) 0.1 (b) 0.1
3. 10/117 = 0.0855
4. (a) 0.0034 (b) 0.0027 (c) 0.7455
5. (a) 22/25 (b) 3/25 (c) 17/50
6. 3/5

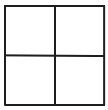
9 - Conditional probability

Example 1. (a) Man has 2 kids. Probability both are boys is:



Answer: Sample space = {BB, BG, GB, GG}. Event of interest = {BB}. So probability is 1/4.

(b) Man has 2 kids. One of them is a boy. Probability both are boys is:



Answer: Sample space = {BG, GB, GG}. Event of interest = {BB}. So probability is 1/3.

Definition. The probability of event E given that event F already happened is:

$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)} = \frac{P(E \cap F)}{P(F)}$$ "probability of E conditioned on F"

If P(F) = 0, P(E|F) is left undefined.

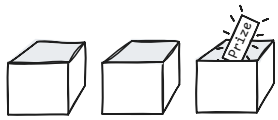
Example 2. A dice is rolled once.

(a) P(Roll a 5) = 1/6

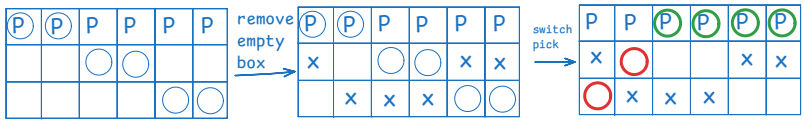
(b) P(Roll a 5 | Roll is even number) P(Roll a 5 and Roll is even)/P(Roll is even)= 0/(1/6) = 0

(c) P(Roll a 5 | Roll is odd number) Answer 1: (By definition)
P(Roll is 5 and roll is odd)/P(roll is odd)
= (1/6) / (1/2) = 2/6.
Answer 2: (Directly) |{5}| / |{1, 3, 5}| = 1/3

Example 3. 1 of 3 boxes has a prize. You pick a box. An empty box you did not pick is removed. To correctly pick the box with the prize, do you switch or not switch boxes?



Not switch: 1/3 chance of winning.
Outcomes for always switch:
(One outcome shown per column)



4/6 chance of win

General multiplication rule:

$$P(E \text{ and } F) = P(E \cap F) = P(F) \cdot P(E|F)$$

Example 4. Poll finds 95% of teens are online, and 60% of online teens use IG. Find probability of teens use IG.

Answer: P(online and use IG) = P(online) x P(use IG | online) = 0.95 x 0.6 = 0.57.
There is about a 57% chance that a randomly selected teen are online and use IG.

Exercises.

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

1. A random sample of 200 adults are classified below by sex and their level of education attained. Find the probability of:
(a) the person is male, given that the person has secondary education;
(b) the person does not have college degree, given that the person is female.

2. Find the probability two dice rolls sum to 8 given that both dice rolls are odd.

3. Find the probability of getting 4 cards from a fresh deck of 52 cards that are only from J's, Q's, K's given that you have only 10's, J's, Q's, K's, A's.

4. A doctor gives correct diagnoses with probability 0.8. Given that the doctor makes an incorrect diagnosis, the probability that the patient sues is 0.9. Find the probability that the doctor misdiagnoses and the patient sues.

Answers:
1 (a) 14/39 (b) 95/112
2. 2/9
3. (12 choose 4)/(20 choose 4)
4. 0.18